

Homework 2 Problems - Version 1

Note: One or two more problems to be added.

2.13, #9: How many bit strings of length 21 have ever 1 followed by 0 and have seventeen 0's and four 1's?

2.14, #16a: Find

$$\sum_{k=0}^n 2^k \binom{n}{k}.$$

2.16, #6: For each of the following r -combinations of [8], find the combination immediately following it in the increasing lexicographical order.

- (a) {1, 3, 5, 6, 8}
- (b) {2, 3, 4, 7}
- (c) {1, 3, 4, 5, 6, 7}
- (d) {4, 5, 6, 7, 8}

2.19, #24: Prove that in a group of at least 2 people, there are always 2 people who have the same number of acquaintances in the group.

2.19, #32: Consider a group of 10 people, each pair of which are either friends or enemies.

- (a) Show that if some person in the group has at least 4 friends, there are 3 people who are mutual friends or 4 people who are mutual enemies.
- (b) Similarly if some person in the group has at least 6 enemies, show that either there are 3 people who are mutual friends or 4 people who are mutual enemies.
- (c) Show that by parts (a) and (b), a group of 10 people, each pair of which are either friends or enemies, has either 3 people who are mutual friends or 4 people who are mutual enemies.

Chapter 2 Additional, #12: Of 15 paint jobs to be done in a day, 5 of them are short, 4 are long, and 6 are of intermediate length. If the 15 jobs are all distinguishable, in how many different orders can they be run so that:

- (a) All the short jobs are run at the beginning?
- (b) All the jobs of the same length are run consecutively?

3.1, #15: Show that in a graph G with n vertices and e edges, there is a vertex of degree at least $\frac{2e}{n}$.