05 July 2009

Homework 3 Problems - Final Version

Note: Many problems in this assignment refer to graphs shown in the book. Please refer to the book for those graphs.

3.2, # 5: For each digraph of Figure 3.7, determine if it is strongly connected.
3.3, # 12: For each graph of Figure 3.50:

(a) Find $\omega(G)$.

(b) Find $\alpha(G)$.

(c) Find $\chi(G)$ and verify the inequality (3.9).

Inequality (3.9) can be found on page 149 and is as follows:

$$\chi(G)\alpha(G) \geq |V(G)|. \tag{3.9}$$

3.3, # 22: *(modified)* Prove that graph (a) of Figure 3.51 is nonplanar using *either* Kuratowski's Theorem of Theorem 3.3.
3.4, # 5: For each of the graphs in Figure 3.65, find the chromatic polynomial using reduction theorems.
3.5, # 6: Give an example of a graph G with $n = e + 1$ but such that G is not a tree.
3.5, # 16: Prove that if any edge is deleted from a tree, the resulting graph will be disconnected.
3.7, # 18: Suppose that a square 0-1 symmetric matrix has 0's on its diagonal. Is it necessarily the adjacency matrix of some graph?

Extra Problem: Give an example of a graph G on 7 vertices where $\omega(G) = 3$ but $\chi(G) = 4$. (It is a fact that for any k, there are graphs with $\omega(G) = 3$ but $\chi(G) = k$.)