**16 July 2009**

# *Quiz 3*

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**Power series:**

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots$$
$$e^x = \sum_{k=0}^{\infty} \frac{1}{k} x^k = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$
$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots$$
$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots$$

1. Find the sequence whose ordinary generating function is given as

$$\frac{x}{1-x^2}.$$

We go step by step:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

(derivative - it's okay to have the sum still start at $k = 0$)
$$\frac{1}{1-x^2} = \sum_{k=0}^{\infty} kx^{k-1}$$

(multiply by $x$)
$$\frac{x}{1-x^2} = \sum_{k=0}^{\infty} kx^k$$

So the sequence is just $a_k = k$, or $(a_k) = (0, 1, 2, 3, \ldots)$.

2. A customer wants to buy six pieces of fruit, including at most two apples, at most two oranges, at most two pears, and at least one but at most two peaches. How many ways are there to buy six pieces of fruit if any two pieces of fruit of the same type are indistinguishable?

The generating function is as follows:

$$(1 + x + x^2)(1 + x + x^2)(1 + x + x^2)(x + x^2).$$

The coefficient of $x^6$ will yield the correct answer (which is 9).

3. How many ways are there to choose $k$ shares of stock if stock is available from four different companies and you must buy at least one share from each company? Express your answer as "the coefficient of ___ in ___" and write your generating function in succinct form (you're not allowed to use ellipses (...) in your expression).

The generating function for this problem is

$$(x + x^2 + x^3 + \ldots)^4 = \left(\frac{x}{1-x}\right)^4.$$

We are looking for the coefficient of $x^k$.